

D-branes in Type IIA and Type IIB theories from tachyon condensation

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ABSTRACT: In this paper we will construct all D-branes in Type IIA and Type IIB theories via tachyon condensation. Then we propose form of Wess-Zumino term for non-BPS D-brane and we will show that tachyon condensation in this term leads to standard Wess-Zumino term for BPS D-brane.

KEYWORDS: D-branes.

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1. Introduction

Non-BPS D-branes have been intensively studied in recent years (see, for example [7, 8, 13, 14, 15] and for review, see [4, 5, 6]). It was proposed in ref. [9, 10, 11] that all D-branes in Type IIA, IIB and Type I theory can be classified via K-theory. This classification is based on tachyon condensation in unstable system of space-time filling branes, D9-branes and antibranes in Type IIB theory and non-BPS D9-branes in Type IIA theory. In this approach, existence of stable BPS D-branes was deduced from topological arguments. It would be nice to see how these branes emerge directly from non-BPS D9-branes in IIA theory or from system D9-branes and antibranes in IIB theory. In this paper we would like to show this phenomena.

In the previous paper [1], we have proposed action for system of non-BPS D9-branes, following [2]. We have shown that via tachyon condensation in form of kink solution on the system of N non-BPS D9-branes we are able to obtain action for $N - k$ D8-branes and k D8-antibranes. Than we have shown that we are able to obtain BPS D6-brane from two D9-branes in Type IIA theory in "step by step" construction, that is based on tachyon condensation in form of kink solution. We have also shown, that the action for D6-brane turns out directly from action for system D8-brane and antibrane from tachyon condensation in the form of vortex solution on world-volume of system brane and antibrane. We have finished the paper [1] with

analysing of Wess-Zumino (WZ) term for non-BPS D-brane and we have discussed some problems related to tachyon condensation in WZ term. in this term.

In this paper we will continue in our previous work of tachyon condensation. The starting point will be action for 16 non-BPS D9-branes in Type IIA theory. The action for non-BPS D-brane was proposed in [2] and we have generalised this action for the system of N non-BPS D-branes in Type IIA theory in [1] in the same way as for ordinary BPS D-brane, see for example [20, 21]. As was explained in [2], action for non-BPS D-brane contains term, which expresses presence of tachyon on the world-volume of non-BPS D-brane. This term has a property, which lies in heart of our construction, that for tachyon equal to its vacuum value, it is zero. We have made some comments about this term in [1], where we have estimated its form on general grounds. In this paper we will see, that with using this simple term, we are able to get some interesting results.

Plan of this paper is follows. In section (2) we will show how our idea works on rather simple example of "step by step" construction of tachyon condensation on world-volume of 16 non-BPS D9-branes in Type IIA theory. We will see, that in this construction we obtain action for 16 D0-branes in Type IIA theory, with agreement with [10], but there is a slight difference with [10], where was argued that due to the tachyon condensation we are able to get one single D0-brane. In fact, as we will see on many examples of tachyon condensation on world-volume of D9-branes in IIA theory, we always get action for 16 D-branes, some of them form a bound state, so they do not contribute to the dynamic of the system, but their presence can be deduced from tension of resulting brane (for example, action that arises from tachyon condensation in section (2) contains factor 2^4 expressing the fact that D0-brane is a bound state of 16 D0-branes). This result is consequence of the fact, that 16 non-BPS D9-branes participate in the construction of D-branes in Type IIA theory.

In section (3) we will discuss construction of general Dp-branes in Type IIA theory via tachyon condensation in generalised vortex solution, following the approach in [10]. Again we obtain correct action for D-branes. In the end of these section, we will show that we are also able to obtain all lower dimensional D-branes in Type IIA theory from system of 16 non-BPS D9-branes which is in agreement with [10]. Again we will see that the resulting action describes 16 D-branes.

In section (4) we turn to the problem of construction of non-BPS D-branes in Type IIA theory, following [17]. We will show that with tachyon solution presented in [17] we are able to obtain action for non-BPS D-brane in Type IIA theory, either with direct construction presented in section (3) or with step by step construction presented in section (2).

In section (5) we will discuss the construction of BPS and non-BPS D-branes in Type IIB theory.

In section (6) we propose form of Wess-Zumino term for non-BPS D-branes. We start from the WZ term for single non-BPS D-brane presented in the ref.[18] and

generalise this result for system of N D-branes and we also propose higher terms in covariant derivative of tachyon, which are needed for correct reproduction of WZ term for ordinary BPS D-brane, as we will see on the example of tachyon condensation in "step by step" construction leading to the D0-brane. Again we will see that resulting charge of D0-brane contains factor 16. The emergence of this factor in WZ term is crucial, because the resulting D-brane should be stable object [9, 10] and such a object should be BPS state of theory and consequently charge and tension of this object must be equal.

Then we will show that tachyon condensation in the form presented in (3) gives the correct value of Wess-Zumino term for BPS D-branes and non-BPS D-branes.

In section (7) we sum up our result and propose other possibilities of our research.

2. Step by step construction

We would like to show, that in our approach we are able to obtain all D-branes in Type IIA, IIB theories. We will start with IIA theory and we will construct D-branes with using tachyon condensation as in [10]. Firstly, we will show "step by step" construction, where we will construct D-branes from kink tachyon solution.

Our starting point is the low energy action for N non-BPS D9-branes:

$$S = - \int d^{10}x \left\{ 1 + \frac{(2\pi\alpha')^2}{4} (\text{Tr} F_{MN} F^{MN} + 2i \text{Tr} \theta_L \Gamma^M D_M \theta_L + \right. \\ \left. + 2i \text{Tr} \theta_R \Gamma^M D_M \theta_R) \right\} F(T, DT, \dots) \quad (2.1)$$

In this section we consider only leading order terms in expansion of DBI action, because there are some problems in generalisation of DBI action for non-Abelian case (for review, see [3]). In this action: $M, N = 0, \dots, 9$, θ_R is right handed Majorana-Weyl spinor, θ_L is left handed Majorana-Weyl spinor and F is a function expressing interaction between massless fields coming from open string sector and tachyon as well as interaction between tachyon and fields coming from closed string sector, graviton, antisymmetric two form ...). In this article, we are interested only in trivial background, so that there are no interaction between tachyon and fields coming from closed string sector. As was argued in paper [1], this term has a form

$$F(T, DT..) = \frac{\sqrt{2}2\pi}{Ng(4\pi^2\alpha')^{\frac{11}{2}}} \left[\text{Tr} D_M T D^M T + \text{Tr}(f(T) \overline{\theta_R} \theta_L) + V(T) \right] \quad (2.2)$$

and tachyon potential has a form [10]:

$$V(T) = -m^2 \text{Tr} T^2 + \lambda \text{Tr} T^4 + \lambda \text{Tr} T_v^4 \quad (2.3)$$

where

$$T_v^2 = \frac{m^2}{2\lambda} \quad (2.4)$$

is minimum of potential and we have included constant term into potential in order to have $V(T = T_0) = 0$. In (2.2) we have included normalisation factor $\frac{1}{N}$ for reason, which will be clear later. We must mention that this potential is zeroth order approximation of potential given in [29, 28]. We take this simple form of potential, because we can get analytic solution of equation of motion for tachyon. Finally we have included in (2.2) factor $\frac{\sqrt{2}2\pi}{g(4\pi^2\alpha')^{\frac{1}{2}}}$, where g is a string coupling constant. This factor corresponds to the tension of non-BPS D9-brane.

We will demonstrate that with using this action, we are able to obtain action for single D0-brane in Type IIA theory. As was proposed in [10], natural gauge group on non-BPS D9-branes is $U(16)$. Now we show "step by step" construction, where in each step we use tachyon kink solution.

Step 1 After variation of action (3.3), we get equation of motion of motion for tachyon fields

$$\frac{\delta F}{\delta T_{ij}} - D_M \left(\frac{\delta F}{\delta D_M T} \right)_{ij} - \partial_M (G) (D^M T)_{ij} = 0 \quad (2.5)$$

where generally $i, j = 0, \dots, 16$ and where the symbol G means the first bracket in (3.3), which includes kinetic terms for all massless fields. We take tachyon solution in the form:

$$T(x) = \begin{pmatrix} T_0(x^9)1_{8 \times 8} & T(y)\delta(x^9) \\ \overline{T(y)}\delta(x^9) & -T_0(x^9)1_{8 \times 8} \end{pmatrix} \quad (2.6)$$

where y means coordinates $x^i, i = 0, \dots, 8$ and delta function in the off-diagonal elements has a formal meaning, which expresses the fact that off-diagonal modes are localised in the core of the vortex. We have also taken T_0 in the form of kink solution of equation:

$$-2 \frac{d^2}{dx^2} T_0(x) + \frac{d}{dT} \mathcal{V}(T) = 0 \quad (2.7)$$

where $\mathcal{V} = -m^2 T^2 + \lambda T^4$. We will see that the tachyon kink solution is nonzero in region of size of string scale, so that in the zero slope limit $\alpha' \rightarrow 0$ reduces to the solution localised in single point $x = 0$. This solution can serve as a justification of approach in [1], where we have taken solution in the form of step function, which appears as a zero slope limit of ordinary kink solution.

Solution of previous equation is ordinary kink solution, which can be found in many books about extended field configurations. We will review the basic facts about this solution.

When we multiply equation (2.7) with T' we get

$$2T''T' = \frac{d\mathcal{V}}{dT}T' \Rightarrow (T')^2 = \mathcal{V} \quad (2.8)$$

when we have made integration over x . From previous equation we get expression

$$\frac{dT}{\sqrt{\mathcal{V}}} = dx \quad (2.9)$$

and integration we get (we take condition that for $x_0 = 0, T_v = 0$):

$$x = \int \frac{dT}{\sqrt{\lambda}(T^2 - T_v^2)} = \frac{1}{\sqrt{\lambda}\sqrt{\frac{m^2}{2\lambda}}} \operatorname{arctangh}\left(\frac{T}{T_v}\right) \quad (2.10)$$

where $T_v^2 = \frac{m^2}{2\lambda}$. From previous equation we get

$$T = T_v \tanh\left(\frac{mx}{\sqrt{2}}\right) \quad (2.11)$$

Its first derivative is equal to

$$T_v \frac{m}{\sqrt{2}} (1 - \tanh^2\left(\frac{mx}{\sqrt{2}}\right))$$

when we put previous results into form of F function we get

$$F = \frac{N2\pi\sqrt{2}}{N(4\pi^2\alpha')^{\frac{11}{2}}g} \left[\frac{T_0^2 m^2}{2} \left(1 - \tanh^2\left(\frac{xm}{\sqrt{2}}\right) \right)^2 + \lambda T_v^4 (\tanh^2\left(\frac{mx}{\sqrt{2}}\right) - 1)^2 \right] \quad (2.12)$$

or equivalently ¹

$$F = \frac{N2\pi\sqrt{2}m^4}{N(4\pi^2\alpha')^{\frac{11}{2}}g2\lambda} (1 - \tanh^2\left(\frac{mx}{\sqrt{2}}\right))^2 \quad (2.13)$$

In previous equations we have denoted $N = 16$. From the behaviour of function $\tanh(x)$ we know that is equal to one almost everywhere except small region which is equal to $(-1, 1)$. From this fact we see that F is zero outside the region of size of string length l_s . In zero slope limit $\alpha' \rightarrow 0 \Rightarrow m = \frac{1}{\alpha'} \rightarrow \infty$ and we see that resulting vortex is localised in the point $x = 0$. In other words, in this limit the F function effectively looks like a delta function. From this reason we have off-diagonal modes of tachyon fields localised in the core of the vortex, which explain the presence of delta function in (2.6). In fact, this delta function has only symbolic meaning, in calculations we will replace this delta

¹In these expressions we do not include the contributions from off-diagonal modes of tachyon. The general expression of function F will be given below.

function by factor $2(1 - \tanh^2(\frac{mx}{2\sqrt{2}}))^2$ that can serve as a regularisation of the delta function as we have seen above.

The next calculation is the same as in [1] and we refer to this paper for more details. After some calculations we get following form of F :

$$F = \frac{2\pi\sqrt{2}}{16(4\pi^2\alpha')^{\frac{11}{2}}g} 2(1 - \tanh^2(\frac{mx}{\sqrt{2}}))^2 \left[\frac{16m^4}{4\lambda} + 2\text{Tr}(XT - TX')(X'\bar{T} - \bar{T}X) + \right. \\ \left. + 2\text{Tr}(\tilde{D}T^\mu \overline{\tilde{D}_\mu T}) + (-m^2\text{Tr}(T\bar{T}) + \lambda\text{Tr}(T\bar{T})^2) + \text{Tr}f(T)\bar{B}\theta + \text{Tr}(f(\bar{T})\bar{C}\theta') \right] \quad (2.14)$$

After putting (2.14) into (2.1) we can easily make integration over x due to the fact that first bracket is independent on x coordinates. This important fact comes from the third term in (2.7), because covariant derivative with respect x is nonzero so that derivation of G with respect to x should be zero, so we get condition that all massless fields are independent on x coordinates. Integration over x gives

$$2 \int_{-\infty}^{\infty} dx \left(1 - \tanh^2(\frac{mx}{\sqrt{2}}) \right)^2 = \frac{8\sqrt{2}}{3m} = \frac{8\sqrt{2}}{6\pi} (4\pi^2\alpha')^{\frac{1}{2}} = 0.606(4\pi^2\alpha')^{\frac{1}{2}} \quad (2.15)$$

We will discuss this numerical value in the end of the section. For the time being we can claim the due to the tachyon condensation in the form of kink solution we have obtained the action for 8 D8-branes and 8 D8-antibranes [1]:

$$S = -0.606(4\pi^2\alpha')^{1/2}C_9 \int_{R^{1,8}} d^9x \left[1 + \frac{(2\pi\alpha')^2}{4} \{ \text{Tr}(F_{\mu\nu}F^{\mu\nu} + 2D_\mu X D^\mu X + \right. \\ \left. + 2i\bar{\theta}(\Gamma^\mu D_\mu \theta + \Gamma^9[X, \theta]) + 4i\text{Tr}(\bar{B}\Gamma^\mu \tilde{D}_\mu B + \bar{B}\Gamma^9(XB - BX')) + \right. \\ \left. + \text{Tr}(F'_{\mu\nu}F'^{\mu\nu} + 2D'_\mu X' D'^\mu X' + 2i\bar{\theta}'(\Gamma^\mu D'_\mu \theta' + \Gamma^9[X', \theta'])) \} \right] \times \\ \times \frac{1}{16} \left\{ 16\frac{m^4}{4\lambda} + 2\text{Tr}(XT - TX')(X'\bar{T} - \bar{T}X) + \right. \\ \left. + 2\text{Tr}(\tilde{D}T^\mu \overline{\tilde{D}_\mu T}) + (-m^2\text{Tr}(T\bar{T}) + \lambda\text{Tr}(T\bar{T})^2) \right. \\ \left. + \text{Tr}f(T)\bar{B}\theta + \text{Tr}(f(\bar{T})\bar{C}\theta') \right\} \quad (2.16)$$

where X, θ, A belong to the adjoint representation of $U(8)$, which corresponds to the gauge group of 8 D8-branes, similarly X, θ', A' correspond to 8 D8-antibranes and $T, B = C^\dagger$ are tachyon and spinor fields respectively coming from the string sector connecting brane and antibrane and they transform in

the $(\mathbf{8}, \bar{\mathbf{8}})$ of $U(8) \times U(8)$. Finally, we also define $DX = dX + [A, X]$, $D'X' = dX' + [A', X']$, $\tilde{D}B = dB + AB - BA'$ and we have used notation $C_p = \frac{2\pi\sqrt{2}}{(4\pi^2\alpha')^{\frac{p+1}{2}}g}$.

Now we proceed to the second step, which is a tachyon condensation in the form of kink solution on the world-volume of these branes.

Step 2 We will construct tachyon kink solution on world-volume 8 D8-branes and 8 D8-antibranes. The solution has a form:

$$T(x^8)_{ij} = T_0(x^8)\delta_{ij} \quad (2.17)$$

where $i, j = 1 \dots 8$ and where $T_0(x^8)$ is solution of (2.7). We will see that this solution place constrains on the form of massless fields.

Firstly, we see, that (2.17) breaks gauge symmetry $U(8) \times U(8)$ into diagonal subgroup $U(8)$. Now we will solve equation of motion in point $x^8 \neq 0$, where tachyon field is in its vacuum value and it is constant, so that equation of motion reduces to the condition:

$$\frac{\delta F}{\delta T} = 0 \quad (2.18)$$

For term with transverse fluctuation, we obtain:

$$\frac{\delta T}{\delta T_{mn}} = (X_{ij}\delta_{mj}\delta_{kn} - \delta_{im}\delta_{jn}X'_{jk})(X'_{kl}\bar{T}_{li} - \bar{T}_{kl}X_{li}) = 0 \quad (2.19)$$

We show, that solution of this equation is condition

$$X_{ij} = X'_{ij} \quad (2.20)$$

When we insert (2.20) into (2.19) and using (2.17), we get:

$$(\dots)(X_{kl} - X'_{kl}) = 0$$

In order to obtain equation of motion for tachyon, we must vary the action (2.16) with respect the tachyon field. As in previous step we obtain the term $\partial_{x^8}(G)\tilde{D}^8T$ where G means the expressions containing massless fields. Due to the fact that \tilde{D}_8T is nonzero we obtain the condition that all massless fields should be independent on x^8 . From this fact we can claim that previous condition (2.20) for fields describing transverse fluctuations hold on the whole axis x^8 .

In the same way we obtain condition

$$A_{ij}^\mu = A_{ij}'^\mu \quad (2.21)$$

where $\mu = 0, \dots, 7$, because kinetic term in (2.16) reduces for constant tachyonic solution to:

$$(A_\mu T - T A'_\mu)(A'^\mu \bar{T} - \bar{T} A^\mu) \quad (2.22)$$

With using the same arguments as for scalar fields, this condition must hold for all x^8 .

Now we proceed to the question of fermionic fields, for which we obtain following equation from varying of F :

$$\text{Tr} \left(g(T_0) \bar{B} \theta + g(T_0) \bar{C} \theta' \right) = 0 \quad (2.23)$$

where $g(T_0) = \frac{dg(T)}{dT} \Big|_{T=T_0}$ and where we have used the fact, that $T_0 = \bar{T}_0$. We write $B = X + iY$, $C = B^\dagger = X - iY$ where we have defined Hermitean matrices X, Y . Then (2.23) has a form:

$$g(T_0) \left[(X - iY) \Gamma^0 \theta + (X + iY) \Gamma^0 \theta' \right] = g(T_0) \left[X \Gamma^0 (\theta + \theta') + iY \Gamma^0 (\theta - \theta') \right] = 0 \quad (2.24)$$

The solution of previous equation is

$$X = 0, \quad \theta = \theta' \quad (2.25)$$

In the same way we could take $Y = 0$, $\theta = -\theta'$, but this result is the same as previous one.

We must also show that in the point x^8 , where tachyon is in its vacuum value, the function F is zero. We know that covariant derivatives, interaction terms with fermionic fields and scalar fields are zero. The remaining terms are

$$16 \frac{m^4}{4\lambda} + 2V(T_v) = 0 \quad (2.26)$$

when we have used $16 \frac{m^4}{4\lambda} = -2V(T_v)$. When we sum up the kinetic terms for tachyon, potential terms together with constant term, we get the expression:

$$16 \frac{m^4}{2\lambda} \left(1 - \tanh^2 \left(\frac{mx}{\sqrt{2}} \right) \right)^2 \quad (2.27)$$

which is the same expression as in the previous step, where the meaning of this formula has been discussed.

In previous part we have obtained number of constrain on the massless fields, that can suggest non-BPS D7-brane. Indeed, the tachyon solution (2.17) is not the most general for describing D7-brane. Remember, that in the kink solution only real part of the tachyon field is fixed. We have than freedom to add to the solution (2.17) an imaginary part, that is function of remaining coordinates x^0, \dots, x^7 (we denote these coordinates as y and this imaginary part is localised

only in the point $x^8 = 0$, because outside this point we would like to have pure vacuum). So that generalised tachyon field is

$$T(x^8, y) = T_0(x^8) + iT(y)\delta(x^8), \quad T(y)^\dagger = T(y) \quad (2.28)$$

where again delta function has symbolic meaning as in previous step. Now we insert this tachyon field into second bracket in (2.16) and we use the constrains for massless fields obtained in previous part, that must hold also for generalised solution, which modifies only behaviour of tachyon field in the core of the kink. Covariant derivative has a form:

$$D_x T = \frac{d}{dx^8} T_0(x^8) + i(A_8 T(y) - T(y) A_8) \delta(x^8) = \frac{d}{dx^8} T_0(x^8) + i[A_8, T(y)] \delta(x^8) \quad (2.29)$$

where we have used $[A, T_0] = 0$. Then we obtain :

$$\text{Tr} D_{x^8} T D^{x^8} \bar{T} = 8 \left(\frac{d}{dx^8} T_0(x^8) \right)^2 + \text{Tr} [X^8, T]^2 \delta(x^8) \quad (2.30)$$

where $X^8 = A^8, T = T(y)$. In the action for brane+antibrane we also have term:

$$\text{Tr} (XT - TX') (X' \bar{T} - \bar{T} X) \quad (2.31)$$

and with using (2.20, 2.28) this term reduces into:

$$\text{Tr} [X^9, T(y)]^2 \delta(x^8) \quad (2.32)$$

Now we take $\mu = 0, \dots, 7$. Then from the remaining covariant derivatives we obtain:

$$D_\mu T(x, y) = i(\partial_\mu T(y) + [A_\mu, T(y)]) \quad (2.33)$$

As a result, we obtain from kinetic term for tachyon and term containing transverse fluctuation (2.32) the final expression:

$$8T'(x^8)^2 + \text{Tr} D_\mu T(y) D^\mu T(y) + \delta_{ij} \text{Tr} [X^i, T] [X^j, T] \delta(x^8) \quad (2.34)$$

where $i, j = 8, 9$ and we have used notation $\frac{d}{dx^8} T_0 = T'_0$.

Potential term has a form

$$V(T) = (-m^2 \text{Tr} T(y)^2 + \lambda \text{Tr} T(y)^4) \delta(x^8) + V(T_0) \quad (2.35)$$

As a last step, fermionic interaction term in point $x^8 = 0, T_0 = 0$, reduces with using (2.25) in

$$\text{Tr} \left(if(T(y)) (-iY) \Gamma^0 \theta - if(T(y)) (iY) \Gamma^0 \theta \right) \delta(x^8) = 2 \text{Tr} f(T(y)) \bar{B} \theta \delta(x^8) \quad (2.36)$$

where we have renamed $Y \rightarrow B$. In previous calculations we have used the fact that $T(y)$ is localised in the point $x^8 = 0$ where $T_0(x^8)$ is equal to zero. As a result, the whole second bracket in (2.16) reduces to:

$$\begin{aligned} & \frac{1}{8} \left(8 \frac{m^4}{4\lambda} + \text{Tr} D_\mu T D^\mu T + \delta_{ij} \text{Tr} [X^i, T] [X^j, T] + \right. \\ & \left. + (-m^2 \text{Tr} T^2 + \lambda \text{Tr} T^4) + \text{Tr} f(T) \overline{B} \theta \right) \times 2 \left(1 - \tanh^2 \left(\frac{x^8 m}{\sqrt{2}} \right) \right)^2 \end{aligned} \quad (2.37)$$

where we have proceed in the same way as in step one.

Now we return to the first bracket in (2.16). With using (2.20, 2.25, 2.21) and the fact, that all fields are independent on x^8 , we obtain the following results:

$$\text{Tr}(F^2 + F'^2) \Rightarrow \text{Tr}(2F_{\mu\nu} F^{\mu\nu} + 4D_\mu X^8 D^\mu X^8) \quad (2.38)$$

where $D_\mu = \partial_\mu + [A_\mu, \cdot]$. In the same way, we obtain for fermionic fields ($\theta = \theta'$):

$$4i \text{Tr} \overline{\theta} (\Gamma^\mu D_\mu \theta + \delta_{ij} \Gamma^i [X^j, \theta]) \quad (2.39)$$

and

$$4i \text{Tr} \overline{B} (\Gamma^\mu D_\mu B + \delta_{ij} \Gamma^i [X^j, B]) \quad (2.40)$$

We see, that all terms have common factor 2. We than obtain the final result:

$$\begin{aligned} S = & -2(0,606)^2 C_7 \int_{R^{1,7}} d^8 x \left[1 + \frac{(2\pi\alpha')^2}{4} \left\{ \text{Tr} F_{\mu\nu} F^{\mu\nu} + 2\delta_{ij} \text{Tr} D_\mu X^i D^\mu X^j + \right. \right. \\ & \left. \left. + 2i \text{Tr} \overline{\theta} (\Gamma^\mu D_\mu \theta + \delta_{ij} \Gamma^i [X^j, \theta]) + 2i \text{Tr} \overline{B} (\Gamma^\mu D_\mu B + \delta_{ij} \Gamma^i [X^j, B]) \right\} \right] \times \\ & \times \frac{1}{8} \left(\text{Tr} D_\mu T D^\mu T + \delta_{ij} \text{Tr} [X^i, T] [X^j, T] + V(T) + \text{Tr} f(T) \overline{B} \theta \right) \end{aligned} \quad (2.41)$$

where we have included constant term $8 \frac{m^4}{4\lambda}$ from (2.34) into (2.35). This new potential has important property, that it is zero for tachyon equal to its vacuum value.

$$V(T) = 8 \frac{m^4}{4\lambda} - \text{Tr} m^2 T^2 + \text{Tr} \lambda T^4 \quad (2.42)$$

We see, that (2.41) is natural action for 8 non-BPS D7-branes in IIA theory. We see the factor 2 in front of the action, which reflects the fact that 16 D9-branes have participated in construction of 8 non-BPS D7-branes.

As a next step, we will construct kink solution on world-volume of this system.

Step 3 We take kink solution in the form:

$$T_0(x^7) = \begin{pmatrix} T_0(x^7)_{14 \times 4} & 0 \\ 0 & -T_0(x^7)_{14 \times 4} \end{pmatrix} \quad (2.43)$$

where $T_0(x^7)$ is the solution of equation (2.7).

The discussion is the same as in step 1, so we briefly recapitulate the result. However, there is one difference. We have term $\text{Tr}[X, T]^2$ in the second bracket in (2.41). Analysis of this term gives the same condition as in the case of gauge fields in step 1, namely X must have a form :

$$X^i = \begin{pmatrix} X^i & 0 \\ 0 & X^i \end{pmatrix} \quad (2.44)$$

where $X^i, X^i \in U(4)$.

With this kink solution, we obtain action describing 4 D6-branes and 4 D6-antibranes, where each D6-brane or antibrane is a bound state of two D6-branes or antibranes respectively. The system has a gauge group $U(4) \times U(4)$. This action is :

$$\begin{aligned} S = -2(0.606)^3 C_6 \int_{R^{1,6}} d^7 x \left[1 + \frac{(2\pi\alpha')^2}{4} \left\{ \text{Tr} \left(F_{\mu\nu} F^{\mu\nu} + 2D_\mu X D^\mu X + 2i\bar{\theta}(\Gamma^\mu D_\mu \theta + \right. \right. \right. \\ \left. \left. + \delta_{ij} \Gamma^i [X^j, \theta] \right) + 4i\text{Tr}(\bar{B}\Gamma^\mu \tilde{D}_\mu B + \bar{B}\delta_{ij} \Gamma^i (X^j B - B X^j)) \right. \\ \left. + \text{Tr} \left(F'_{\mu\nu} F'^{\mu\nu} + 2D'_\mu X' D'^\mu X' + 2i\bar{\theta}'(\Gamma^\mu D'_\mu \theta' + \delta_{ij} \Gamma^i [X'^j, \theta']) \right) \right\} \times \\ \times \frac{1}{8} \left\{ 8\frac{m^4}{4\lambda} + 2\delta_{ij} \text{Tr}(X^i T - T X^i)(X'^j \bar{T} - \bar{T} X'^j) + \right. \\ \left. + 2\text{Tr}(\tilde{D} T^\mu \overline{\tilde{D}_\mu T} + V(T, \bar{T})) \right. \\ \left. + \text{Tr}(f(T)\bar{B}\theta) + \text{Tr}(f(\bar{T})\bar{C}\theta') \right\} \end{aligned} \quad (2.45)$$

where $\mu, \nu = 0, \dots, 7; i, j = 9, 8, 7$ and trace goes over adjoint representation of $U(4)$ and the meaning of various fields is the same as in step 1. Now we proceed to the step 4.

Step 4 In this step we construct kink solution on world-volume of 8 D6-branes and 8 D6-antibranes. This kink solution is the same as in (2.17), which breaks gauge symmetry $U(4) \times U(4)$ into its diagonal subgroup $U(4)$. As a result, we obtain action for 4 non-BPS D5 branes in IIA theory, where each D-brane is a bound state of four D-branes:

$$\begin{aligned} S = -4(0.606)^4 C_5 \int_{R^{1,5}} d^6 x \left[1 + \frac{(2\pi\alpha')^2}{4} \left\{ \text{Tr} F_{\mu\nu} F^{\mu\nu} + 2\delta_{ij} \text{Tr} D_\mu X^i D^\mu X^j + \right. \right. \\ \left. + 2i\text{Tr} \bar{\theta}(\Gamma^\mu D_\mu \theta + \delta_{ij} \Gamma^i [X^j, \theta]) + 2i\text{Tr} \bar{B}(\Gamma^\mu D_\mu B + \delta_{ij} \Gamma^i [X^j, B]) \right\} \times \\ \times \frac{1}{4} \left(\text{Tr} D_\mu T D^\mu T + \delta_{ij} \text{Tr} [X^i, T][X^j, T] + V(T) + \text{Tr} f(T)\bar{B}\theta \right) \end{aligned} \quad (2.46)$$

where $\mu, \nu = 0, \dots, 5; i, j = 6, \dots, 9$. Now we are going to the next step.

Step 5 Again we construct kink solution on world-volume of four non-BPS D5-branes and as a result, we obtain action for two D4-branes and two D4-antibranes (As in previous parts, each D4-brane is a bound state of four D4-brane, and each D4-antibrane is a bound state of four D4-antibranes):

$$\begin{aligned}
S = -4(0.606)^5 C_4 \int_{R^{1,4}} d^5x \left[1 + \frac{(2\pi\alpha')^2}{4} \left\{ \text{Tr} \left(F_{\mu\nu} F^{\mu\nu} + 2D_\mu X D^\mu X + 2i\bar{\theta}(\Gamma^\mu D_\mu \theta + \right. \right. \right. \\
\left. \left. + \delta_{ij} \Gamma^i [X^j, \theta] \right) + 4i\text{Tr}(\bar{B}\Gamma^\mu \tilde{D}_\mu B + \bar{B}\delta_{ij}\Gamma^i (X^j B - B X'^j)) + \right. \\
\left. + \text{Tr} \left(F'_{\mu\nu} F'^{\mu\nu} + 2D_\mu X' D^\mu X' + 2i\bar{\theta}'(\Gamma^\mu D'_\mu \theta' + \delta_{ij}\Gamma^i [X'^j, \theta']) \right) \right\} \times \\
\times \frac{1}{4} \left\{ 4\frac{m^4}{4\lambda} + 2\delta_{ij}\text{Tr}(X^i T - T X'^i)(X'^j \bar{T} - \bar{T} X^j) + \right. \\
\left. + 2\text{Tr}(\tilde{D}T^\mu \bar{\tilde{D}}_\mu T + V(T, \bar{T})) \right. \\
\left. + \text{Tr}(f(T)\bar{B}\theta) + \text{Tr}(f(\bar{T})\bar{C}\theta') \right\}
\end{aligned} \tag{2.47}$$

where $\mu, \nu = 0, \dots, 4; i, j = 5, \dots, 9$. Now we arrive to the next step.

Step 6 We construct kink solution on world-volume of 2 D4-branes and 2D4-antibranes. Again, this solution breaks symmetry $U(2) \times U(2)$ into diagonal subgroup $U(2)$. As a result, we obtain action for two non-BPS D3-branes in IIA theory:

$$\begin{aligned}
S = -8(0.606)^6 C_3 \int_{R^{1,3}} d^4x \left[1 + \frac{(2\pi\alpha')^2}{4} \left\{ \text{Tr} F_{\mu\nu} F^{\mu\nu} + 2\delta_{ij}\text{Tr} D_\mu X^i D^\mu X^j + \right. \right. \\
\left. + 2i\text{Tr}\bar{\theta}(\Gamma^\mu D_\mu \theta + \delta_{ij}\Gamma^i [X^j, \theta]) + 2i\text{Tr}\bar{B}(\Gamma^\mu D_\mu B + \delta_{ij}\Gamma^i [X^j, B]) \right\} \times \\
\times \frac{1}{2} \left(\text{Tr} D_\mu T D^\mu T + \delta_{ij}\text{Tr}[X^i, T][X^j, T] + V(T) + \text{Tr} f(T)\bar{B}\theta \right)
\end{aligned} \tag{2.48}$$

where $\mu, \nu = 0, \dots, 5; i, j = 6, \dots, 9$. Again we must mention, that each D3-brane is a bound state of 8 D3-branes.

Step 7 We construct kink solution on world-volume of two non-BPS D3-branes in IIA theory, which leads to system of D2-brane and D2-antibrane. This solution breaks gauge group $U(2)$ into $U(1) \times U(1)$. As a result, all commutators are zero and we must replace all covariant derivatives with ordinary derivatives. We than obtain action for D2-brane and D2-antibrane:

$$\begin{aligned}
S = -8(0.606)^7 C_2 \int_{R^{1,2}} d^3x \left[1 + \frac{(2\pi\alpha')^2}{4} \left\{ F_{\mu\nu} F^{\mu\nu} + 2\delta_{ij}\partial_\mu X^i \partial^\mu X^j + 2i\bar{\theta}\Gamma^\mu \partial_\mu \theta + \right. \right. \\
\left. + 4i(\bar{B}\Gamma^\mu \tilde{D}_\mu B + \bar{B}\delta_{ij}\Gamma^i (X^j B - B X'^j)) + \right. \\
\left. + (F'_{\mu\nu} F'^{\mu\nu} + 2\delta_{ij}\partial_\mu X'^i \partial^\mu X'^j + 2i\bar{\theta}'\Gamma^\mu \partial_\mu \theta') \right\} \times
\end{aligned}$$

$$\begin{aligned}
& \times \frac{1}{2} \left\{ 2 \frac{m^4}{4\lambda} + 2\delta_{ij} \text{Tr}(X^i T - T X^i)(X'^j \bar{T} - \bar{T} X^j) + \right. \\
& \left. + 2\text{Tr}(\tilde{D}T^\mu \overline{\tilde{D}_\mu T} + V(T, \bar{T})) + \text{Tr}(f(T)\bar{B}\theta) + \text{Tr}(f(\bar{T})\bar{C}\theta') \right\}
\end{aligned} \tag{2.49}$$

where $\mu, \nu = 0, \dots, 2; i, j = 3, \dots, 9$.

Step 8 We consider kink solution on world-volume of D2-brane and D2-antibrane, which leads to the action for one non-BPS D1-brane:

$$\begin{aligned}
S = -16(0.606)^8 C_1 \int_{R^{1,1}} d^2x \left[1 + \frac{(2\pi\alpha')^2}{4} \left\{ F_{\mu\nu} F^{\mu\nu} + 2\delta_{ij} \partial_\mu X^i \partial^\mu X^j + \right. \right. \\
\left. \left. + 2i\bar{\theta}\Gamma^\mu \partial_\mu \theta + 2i\bar{B}\Gamma^\mu \partial_\mu B \right\} \right] \times \\
\times \left(\partial_\mu T \partial^\mu T + V(T) + f(T)\bar{B}\theta \right)
\end{aligned} \tag{2.50}$$

where $\mu, \nu = 0, 1, i, j = 2, \dots, 9$

Finally, we construct tachyon solution on world-volume of non-BPS D1-brane. Then, following [1], the second bracket reduces to the form

$$2(1 - \tanh^2 \left(\frac{mx}{\sqrt{2}} \right))^2 V(T_v) \tag{2.51}$$

and integration of previous expressions gives

$$0.606(4\pi^2\alpha')^{1/2} V(T_v) \tag{2.52}$$

Now we use the results from [29, 28], where it was shown that vacuum value of potential is equal to 0.60 of value of the mass of non-BPS D-brane. Since tachyon potential $v(T)$ presented in ref.[28] is related to our potential with rescaling $V(T) = 2v(T)$, because in normalisation of kinetic term in our action the factor 1/2 is missing, we can claim that vacuum value of potential is equal to

$$V(T_v) = 1.2 \frac{1}{\sqrt{2}}$$

than previous expressions leads to

$$(4\pi^2\alpha')^{1/2} 0.606 V(T_v) = (0.73)(4\pi^2\alpha')^{1/2} \frac{1}{\sqrt{2}} \tag{2.53}$$

we see that tension of D-brane that arises from single tachyon kink is about 0.73 of tension for D-brane, which is in agreement with result [28]. Of course, for D0-brane the resulting tension is much smaller than expected answer, which is result of our rough approximation. We can expect on the grounds given in

ref.[28], that further correction in the tachyon potential will lead to correct results.

To sum up, In the first bracket the fermion field B is identically zero and we finish with action for D0-brane in type IIA theory (more precisely, we end with action for 16 D0-branes in IIA theory, that form a bound state, but we will discuss this issue later), with completely agreement with [10]

$$S = -16(0,606)^9 1.2 \frac{2\pi}{(4\pi^2\alpha')^{1.2}g} \int dt \left[1 + \frac{(2\pi\alpha')^2}{4} (2\partial_t X^i \partial_t X^i + 2i\bar{\theta}\Gamma^0 \partial_t \theta) \right] \quad (2.54)$$

We see, that with this "step by step" construction we are able to obtain all D-branes in IIA theory, and when we start with system D9-branes and D9-antibranes, following [9], we are able to construct all D9-branes in IIB theory as well. However, we would like to see, whether direct construction, presented in [9, 10], can be applied in this approach. We return to this question in next section.

3. Direct construction

In this section we show that we can construct lower dimensional BPS D-brane also directly following [10], where was argued that D-brane of codimension $2k+1$ can be construct as vortex solution on world-volume of 2^k non-BPS D9-branes with gauge group $U(2^k)$. In region around point $x=0$, the tachyon field looks like:

$$T(x) = \sum_{i=1}^{2k+1} \Gamma_i x^i \quad (3.1)$$

where Γ_i are Gamma matrices of group $SO(2k+1)$, which is a symmetry group of transverse space to D(8-2k)-brane and $x^i, i=1, \dots, 2k+1$ are coordinates on this transverse space.

It was argued [10] that this tachyon condensation is equivalent to condensation of tachyon in step by step construction, where tachyon forms a kink solution in each step. In order to obtain correct kink solution, we generalise previous equation in the form

$$T = \sum_{i=1}^{2k+1} \Gamma_i T_i(x^i) \quad (3.2)$$

we will see that tachyon condensation in the form of this field is equivalent to condensation of tachyon in form of step by step construction. We start to solve the equation of motion for tachyon. We write the action for non-BPS D-brane:

$$S = -C_9 \int d^{10}x \left\{ 1 + \frac{(2\pi\alpha')^2}{4} (\text{Tr} F_{MN} F^{MN} + 2i \text{Tr} \theta_L \Gamma^M D_M \theta_L + \right.$$

$$+2i\text{Tr}\theta_R\Gamma^M D_M\theta_R)\} F(T, DT, \dots) \quad (3.3)$$

where $C_p = \frac{2\pi\sqrt{2}}{g(4\pi^2\alpha')^{\frac{p+1}{2}}}$ and we implicitly work in limit $\alpha' \rightarrow 0$, because then we can neglect the higher terms in expansion of Born-Infeld action. In previous expression the F function express integration between tachyon and other fields and has a form:

$$F(T, DT..) = \frac{1}{2^k} [\text{Tr} D_M T D^M T + \text{Tr}(f(T)\overline{\theta}_R\theta_L) + V(T)] \quad (3.4)$$

and V is a potential for tachyon (we again work in zeroth approximation, which allows analytic solution [28]).

$$V(T) = -\text{Tr} m^2 T^2 + \lambda \text{Tr} T^4 + \text{Tr} T_v^4 \quad (3.5)$$

where $T_v = \frac{m^2}{2\lambda}$ is a vacuum value of tachyon field. Equation of motion for tachyon field has a form:

$$-\partial_M(G)D^M T + G(-2D_M D^M T + \frac{\partial U}{\partial T}) = 0 \quad (3.6)$$

where G means the first bracket in action for non-BPS D-brane. We know that tachyon is a function only $2k+1$ transverse coordinates so that around the core we have $D_i T \neq 0$ so that in order to obey equation of motion we must pose the requirement that all fields, which are present in G should be independent on transverse coordinates. We will see that this is natural requirement because the effective size of the core is of order string scale and we know that BI action is valid only for slowly varying fields so that these fields do not change in region of size of string scale. In solving previous equation we must also demand the vanishing covariant derivative with tangent direction to the vortex otherwise we should take $\partial_M G = 0$ for all M and we do not get any interesting dynamical system. From the condition of vanishing the covariant derivative $D_\mu T$, $\mu = 0, \dots, 8-2k$ we get condition on gauge field:

$$D_\mu T = [A_\mu, T] = 0 \Rightarrow A_\mu \in SU(2^k) = 0 \quad (3.7)$$

and only $\mathcal{A}_\mu \in U(1)$ remains as a free dynamical field.

Suppose now that we are in the region out of the core where tachyon is in its vacuum value. Then its derivative is zero and we get the same condition for gauge fields as in previous case:

$$DT_i = 0 = [A_i, T] = 0 \Rightarrow A_i = 0 \quad (3.8)$$

and only $\mathcal{A}_i \in U(1)$ remains undetermined free dynamical field. This condition hold almost on the whole plane and from the previous result, which says that massless fields are not functions of transverse coordinates, we get condition that $A_i \in SU(2^k)$

is zero everywhere. With using these facts the covariant derivative for tachyon reduces to ordinary derivative and variation of kinetic term gives (in the following we write $T_i(x^i) = T_i$:

$$\text{Tr}\delta(d_M(\Gamma_i T^i d^M(\Gamma_j T^j) \Rightarrow -\text{Tr}2\delta T^i \Gamma_i d_M d^M(\Gamma_j T^j) = -22^k \delta T_i d_M d^M T_i \quad (3.9)$$

where we have used $\text{Tr}\Gamma_i \Gamma_j = 2^k \delta_{ij}$. The variation of potential term gives

$$\begin{aligned} \delta V &= \delta T^i (-2m \text{Tr}\Gamma_i \Gamma^j T_j + 4\lambda \text{Tr}\Gamma_i \Gamma^j \Gamma^k \Gamma^l T_j T_k T_l) \\ &= \delta T_i (-2m T^i + 4\lambda T_i (T_j T^j)) \end{aligned} \quad (3.10)$$

and variation of interaction term between fermions and tachyon gives

$$\begin{aligned} \text{Tr}(\delta(a_1 T + a_3 T^3 + \dots) \bar{\theta}_R \theta_L) &= \delta T_i \text{Tr}\Gamma^i (a_1 + a_3 \Gamma^i \Gamma^j T_i T_j + \dots) \bar{\theta}_R \theta_L = \\ &= \delta T_i \text{Tr}\Gamma^i \frac{df(T^2)}{dT} \bar{\theta}_R \theta_L \end{aligned} \quad (3.11)$$

where $T^2 = T_i T^i$.

Now equation of motion have a form:

$$-2d_i d^i T^i + (-2m^2 T_i + 4\lambda T_i^3 + 4\lambda T_i (\sum_{j \neq i} T_j T_j)) + \text{Tr}(\Gamma_i \frac{df}{dT} \bar{\theta}_R \theta_L) = 0; i = 1, \dots, 2k+1 \quad (3.12)$$

In previous equation we do not sum over i and we have used the fact that $d_M T_i = \delta_M^i \partial_i T(x^i)$. Since T_i are all independent, we will see that solution of motion of these equations leads to the kink solutions for all T_i with additional conditions, which must vanish separately. Firstly, we must pose the condition:

$$T_i (\sum_{j \neq i} T^j T^j) = 0 \quad (3.13)$$

Outside the core of the vortex, we have $T_i \neq 0$ so that we must have T_j to be zero. In the point $x^i = 0$ we have solution $T(x^i) = 0$ so that $T(x^j)$ should be nonzero. But this is nothing else than tachyon condensation in the form of step by step construction, where each resulting tachyon is localised on world-volume of non-BPS D-brane or system of branes and antibranes that arises from tachyon condensation in the previous step. The previous condition has the same meaning, but now we do not prefer some particular direction where we start the tachyon condensation. In other words, this tachyon condensation is naturally transversally invariant. With the requirement that fermionic term should be zero separately we get the equation for each T_i in the form:

$$-2d_i d^i T(x^i) + \frac{dU(T(x^i))}{dT} = 0 \quad (3.14)$$

which has a natural solution in the form of kink solution as we have seen in the previous section. The form of the kink solution has a form:

$$T(x^i) = T_v \tanh\left(\frac{mx^i}{\sqrt{2}}\right) \quad (3.15)$$

Now we put $T = \sum_{i=1}^{2k+1} \Gamma_i T^i$ into the F function. At this point we must be more careful, because we know that T_i live only in the point $x^j = 0, j \neq i$. We will have this fact in the mind when we put previous equation into F function and in resulting expression we multiply each term, that is a function of T_i only, with the factor of convergence, which will have properties of delta function and will express the above condition.

$$\text{Tr} d_M(\Gamma_i T^i) d^M(\Gamma_j T^j) = 2^k \sum_{i=1}^{2k+1} \partial_i T^i \partial_i T^i \quad (3.16)$$

and

$$V(T) = -\text{Tr} m^2 (T^i T^j \Gamma_i \Gamma_j) + \lambda \text{Tr} (\Gamma_i \Gamma_j \Gamma_k \Gamma_l T^i T^j T^k T^l) = 2^k \sum_{i=1}^{2k+1} (-m^2 T_i^2 + \lambda T_i^4) \quad (3.17)$$

where we have used the fact that expression $T_i T^j, i \neq j$ is zero from arguments presented above. Then we obtain the form of F function (We use the fact that fermionic terms is zero as will be shown in a moment):

$$F = \left[\left(\left(\frac{dT_1}{dx^1} \right)^2 + (-m^2 T_1^2 + \lambda T_1^4) + \frac{m^4}{4\lambda} \right) + \sum_{i=2}^{2k+1} F_i \right] \quad (3.18)$$

where F_i have a form

$$F_i = \left(\frac{dT_i}{dx^i} \right)^2 - m^2 T_i^2 + \lambda T_i^4 \quad (3.19)$$

We see that we have the similar result as in the step by step construction. The first term in (3.18) has the form

$$\frac{m^4}{2\lambda} \left(1 - \tanh^2 \left(\frac{mx_i}{\sqrt{2}} \right) \right)^2 \quad (3.20)$$

Since we know that all $F_i, i \neq 1$ are localised in the point $x^1 = 0$ we multiply the second term in (3.18) with the factor $2(1 - \tanh^2(\frac{mx}{\sqrt{2}}))^2$ as in previous section. Then we can make integration over x^1 leading to the result $(0,606)(4\pi^2\alpha')^{1/2}$ in front of F function and to the emergence of constant term $V(T_v)$ in F function (we have again used the fact that all fields in the first bracket in (3.3) are independent on x^1). After repeating this calculation the F function will give the contribution $(0.606)^{2k+1} 1.2 \frac{1}{\sqrt{2}} (4\pi^2\alpha')^{\frac{2k+1}{2}}$.

In previous part we have anticipated that fermionic terms is equal to zero. In this paragraph we show that this is really true. We expand the fermionic fields in following way ²:

$$\theta_{L,R} = \theta_{L,R}^0 + \Gamma_i \theta_{L,R}^i \quad (3.21)$$

and after putting this expansion into expression $\text{Tr} \Gamma^i \bar{\theta}_R \theta_L$ we get the equations (we have used the fact that $\frac{df}{dT}$ is nonzero. In the following we will not write this factor):

$$\begin{aligned} \text{Tr}(\Gamma_i \Gamma_k \Gamma_l) \bar{\theta}_R^k \theta_L^l &= 0 \\ \text{Tr}(\Gamma_i \Gamma_l) \bar{\theta}_R^l \theta_L^0 &= 2^k \bar{\theta}_R^i \theta_L^0 = 0 \\ 2^k \bar{\theta}_R^0 \theta_L^i &= 0 \end{aligned} \quad (3.22)$$

$$(3.23)$$

Solution of the last equation is $\bar{\theta}_R^0 = 0$ or $\theta_L^i = 0$, but from the fact that the first equation for $i = k = l$ gives condition $\bar{\theta}_R^i \theta_L^i = 0$, which can be solved as $\theta_L^i = 0$, we see that we must take as a solution of last equation the condition $\theta_R^0 = 0$ and solution of the second equation as a $\theta_R^i = 0$.

In other words we get the result that spinor field θ_R completely disappears as it should for restoring the BPS D-brane. We also see that only $U(1)$ parts of θ_L remains as a free dynamical field which is in agreement with the number of bosonic degrees of freedom. The vanishing of $\theta_L \in SU(2^k)$ can be view as a consequence of vanishing of $A \in SU(2^k)$, since both fields are related through supersymmetric transformations of nonlinearly realised supersymmetry.

Now we can sum the results. From the fact that all fields in the adjoint representation of $SU(2^k)$ and spinor θ_R is zero we obtain the action for Dp-brane of codimension $2k+1$, when we use the fact that all fields are independent on transverse coordinates:

$$S = -2^k (0.606)^{2k+1} 1.2 T_p \int_{R^{1,8-2k}} d^{p+1}x \left[1 + \frac{(2\pi\alpha')^2}{4} (F_{\mu\nu} F^{\mu\nu} + 2\partial_\mu X^i \partial^\mu X^i + 2i\bar{\theta} \Gamma^\mu \partial_\mu \theta) \right] \quad (3.24)$$

where $p = 8 - 2k$ and $T_p = \frac{2\pi}{g(4\pi^2\alpha')^{\frac{p+1}{2}}}$. In previous action we have used

$$F_{i\mu} F^{i\mu} = 2\partial_\mu A_i \partial^\mu A_i = 2\partial_\mu X^i \partial^\mu X^i \quad (3.25)$$

For D0-branes, $k = 4$ and we obtain the same result as in previous section. Again we see the presence of factor 2^k in front of the action, which suggests that resulting configuration corresponds to the bound state of 2^k Dp-branes. We will return to this issue in the end of this section.

²In fact, we should consider more general expansion of θ in the form of completely antisymmetric basic. However it is easy to see that higher terms in this expansion are identically zero. Consider for example $\theta_{R,L}^{ij} \Gamma_{ij}$. Then we obtain two conditions $:\text{Tr} \Gamma_i \Gamma_{kl} \bar{\theta}_R^{kl} \theta_L^0 = 0, \text{Tr} \Gamma_i \Gamma_{kl} \Gamma_{mn} \bar{\theta}_R^{kl} \theta_L^{mn} = 0$. Then the second equation gives condition $\theta_L^{mn} = 0$ and the first equation gives condition $\theta_R^{kl} = 0$. This arguments holds for higher terms as well.

It is also important that with 16 non BPS D9-branes, we are able to construct lower dimensional brane as well. Consider general D-brane of codimension $2k + 1$. As was explained in ref.[10], this brane can be constructed from 2^k non-BPS D9-branes with gauge group $U(2^k)$, where tachyon $t(x) \in U(2^k)$ is a function of $2k + 1$ coordinates and its form is the same as in (3.2). We can put this tachyon field into $U(16)$ as follows

$$T(x) = t(x) \otimes 1 \quad (3.26)$$

where 1 is $2^{4-k} \times 2^{4-k}$ unit matrix. We can take gauge field in the form

$$A = \mathbf{A} * 1 \otimes 1 + A \otimes 1 + 1 \otimes A \quad (3.27)$$

where \mathbf{A} is $U(1)$ part of gauge field, $A \in SU(2^k)$, $A \in SU(2^{4-k})$ and 1 is $2^k \times 2^k$ unit matrix. Then we can proceed as in previous part, because it is easy to see, that only A is fixed with tachyon solution (it is equal to zero) due to the fact that

$$[t(x) \otimes 1, 1 \otimes A] = 0 \quad (3.28)$$

so that A and \mathbf{A} are not fixed with tachyon solution, because do not appear in covariant derivative DT . Then it is also clear, that

$$F = F \otimes 1 + 1 \otimes F + \mathbf{F} * 1 \otimes 1 \quad (3.29)$$

where F is a field strength for A , F is a field strength for A , F is a field strength for A and \mathbf{F} is a field strength for \mathbf{A} . Then kinetic term F^2 reduces into $(\text{Tr}(A \otimes B) = \text{Tr}(A)\text{Tr}(B))$

$$\text{Tr}(F^2) = \text{Tr}(F^2)\text{Tr}(1) + 2\text{Tr}F\text{Tr}F + 2\mathbf{F}(\text{Tr}F\text{Tr}1 + \text{Tr}1\text{Tr}F) + \text{Tr}1\text{Tr}(F^2) + \mathbf{F}\text{Tr}1\text{Tr}1 \quad (3.30)$$

We can immediately see, that second and third term vanishes due to the fact, that $\text{Tr}F = \text{Tr}F = 0$. When we combine \mathbf{F} with F into one single field \mathcal{F} in adjoint representation of $U(2^{4-k})$ and use the fact, that all \mathcal{A} are not function of $2k + 1$ coordinates, the kinetic term reduces into

$$2^k(\text{Tr}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + 2\text{Tr}D_\mu\mathcal{X}^i D^\mu\mathcal{X}^i) \quad (3.31)$$

where 2^k comes from $\text{Tr}1$ and $\mathcal{X}^i, i = 9 - 2k, \dots, 9$ are dynamical fields describing transverse fluctuation of 2^{4-k} D-branes of codimension $2k + 1$ and we have also defined $D\mathcal{X} = d\mathcal{X} + [\mathcal{A}, \mathcal{X}]$.

Analysis of fermions is the same as in case of D0-brane. Again, θ_R is zero and we write θ_L in the same way as A

$$\theta_L = \Theta_L * 1 \otimes 1 + \tilde{\theta}_L \otimes 1 + 1 \otimes \phi_L \quad (3.32)$$

where the Θ_L is $U(1)$ part of θ_L , $\tilde{\theta}_L \in SU(2^k)$ and $\phi \in SU(2^{4-k})$. Again $\theta_R, \tilde{\theta}_L$ are equal to zero thanks to form of tachyon field $T \sim t \otimes 1$. Nonzero dynamical fields

are Θ_L, ϕ and as in case of gauge field we combine Θ with ϕ into one single massless fermionic field $\theta \in U(2^{4-k})$, which has a kinetic and interaction term in resulting action for D-brane

$$2i\text{Tr}\bar{\theta}_L\Gamma^M D_M\theta_L \Rightarrow 2^k(2i\text{Tr}\bar{\theta}\Gamma^\mu D_\mu\theta + 2i\text{Tr}\bar{\theta}\Gamma^i[\mathcal{X}^i, \theta]) \quad (3.33)$$

Finally, with using the fact that F function gives the same contribution as before, we obtain the action

$$S = -2^k(0.606)^{2k+1}1.2T_p \int_{R^{1,8-2k}} d^{p+1}x \left[1 + \frac{(2\pi\alpha')^2}{4}(\text{Tr}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + 2\text{Tr}D_\mu\mathcal{X}^i D^\mu\mathcal{X}^i + 2i\text{Tr}\bar{\theta}\Gamma^\mu D_\mu\theta + 2i\text{Tr}\bar{\theta}\Gamma^i[\mathcal{X}^i, \theta]) \right] \quad (3.34)$$

This action describes 2^{4-k} Dp-branes of codimension $2k+1$, where each D-brane is a bound state of 2^k D-branes of the same codimension, which can be seen from the factor 2^k in front of the action. These results can also be seen in "step by step" construction. Consider, for example, "step by step" construction for D6-brane. This can be schematically written as:

$$U(16) \xrightarrow{\text{kink}} U(8) \times U(8) \xrightarrow{\text{kink},2} U(8) \xrightarrow{\text{kink}} U(8) \quad (3.35)$$

where factor on the second arrow expresses the presence of factor two in front of the action. This sequence correspond to the sequence of branes

$$16D9 \rightarrow 8D8 + 8\overline{D8} \rightarrow 8D7 \rightarrow 8D6$$

Which implies, that this configuration describes system of 8 D6-branes with gauge group $U(8)$ (In fact, as was explained in previous section, each D6-brane is a bound state of two D6-branes). The same "step by step" construction can be used for other D-branes. For D4-brane, we have sequence:

$$U(16) \xrightarrow{\text{kin}} U(8) \times U(8) \xrightarrow{\text{kink},2} U(8) \xrightarrow{\text{kink}} U(4) \times U(4) \xrightarrow{\text{kink},4} U(4) \xrightarrow{\text{kink}} U(4) \quad (3.36)$$

which corresponds to emergence of action for 16 D4-branes with gauge group $U(4)$, with agreement with general result given in (3.34)

To sum up, we have seen, that from configuration of 16 non-BPS D9-branes in IIA theory we can construct all BPS D-branes in IIA theory. In fact, we obtain after appropriate tachyon condensation action, that describes 16 D-branes. The question remains, whether we can describe one single D-brane in this theory. Firstly, we can go into the Coulomb branch of the resulting action and consider one separate D-brane taking the other branes to infinity. Then we obtain action for single D-brane of codimension $2k+1$, but with additional factor 2^k in front of the action, which

suggests, that this brane is a bound state of 2^k D-branes. It is clear, that resulting D-brane looks like ordinary D-brane of codimension $2k+1$, but its tension is different, so we cannot say, that this D-brane is elementary D-brane. We will see the same problem in analysing of WZ term in section (6). At present, we do not know, how we could obtain action for single elementary D-brane. It is possible that clue to this issue lies in more general construction of tachyon condensation corresponding to the D-branes which do not coincide.

4. Non-BPS D-branes in Type IIA theory

In this section we will construct non-BPS D-branes in Type IIA theory from tachyon condensation in the system of N non-BPS D9-branes, following [17], where tachyon configuration for construction of non-BPS D-brane of codimension $2k$ was proposed in the form:

$$T = \sum_{i=1}^{2k} \Gamma_i x^i \quad (4.1)$$

where gamma matrices form a spinor representation of transverse space. We will see that we can construct this non-BPS D-brane with almost any effort, because this construction is directly related to the construction presented in previous section. As in previous section we start with system 2^k non-BPS D9-branes with gauge group $U(2^k)$ on their world-volume.

We generalise (4.1) to the expression:

$$T = \sum_{i=1}^{2k} \Gamma_i T(x^i) \quad (4.2)$$

which has the same form as in case of BPS D-brane. In fact, the analysis of equation of motion for tachyon is the same in both situations (BPS and non-BPS) in case of bosonic terms. Again tachyon condensation in this form will leads to the D-brane of codimension $2k$ with $U(1)$ gauge symmetry on its world-volume. But there is an difference in the case of fermionic terms. Again we have the condition:

$$\Gamma^i \bar{\theta}_R \theta_L = 0, i = 1, \dots, 2k \quad (4.3)$$

but now we must expand the fermionic fields in the form

$$\theta_{L,R} = \sum_{i=1}^{2k+1} \theta_{L,R}^i \Gamma_i \quad (4.4)$$

because we cannot discard the term proportional to Γ^{2k+1} matrix. As in case of BPS D-brane we should make more general expansion with higher antisymmetric combinations of gamma matrix but it can be shown that these higher terms should

vanish in order to obey previous equation. When we put previous expansion of fermions into (4.3), we get the same conditions as in case of BPS D-brane:

$$\begin{aligned}\text{Tr}(\Gamma^i \Gamma^k \Gamma^l) \overline{\theta}_R^k \theta_L^l &= 0 \\ \overline{\theta}_R^i \theta_L^0 &= 0 \\ \overline{\theta}_R^0 \theta_L^i &= 0\end{aligned}\tag{4.5}$$

First equation leads to condition $\theta_L^i = 0, i = 1, \dots, 2k + 1$ the second one to $\theta_R^i = 0, i = 1, \dots, 2k$ and the last equation to condition $\theta_R^0 = 0$. It is important to stress that we have not any condition on θ_R^{2k+1} due to the trivial identity $\text{Tr} \Gamma_i \Gamma_{2k+1} = 0, i = 1, \dots, 2k$. We than see that we have two dynamical fermionic fields θ_R, θ_L , which is a appropriate number of fermionic degrees of freedom for non-BPS D-brane. In fact, there is also one additional tachyon mode related to the matrix Γ^{2k+1} , which is again free dynamical field. From that reason we must put into F function in the (2.16) the form of tachyon field

$$T = T_{cs} + \mathcal{T}(y) \Gamma^{2k+1}\tag{4.6}$$

where T_{cs} is a classical solution of equation of motion, which explicitly form was given above and $\mathcal{T}(y)$ is a free tachyonic field, which is localised on world-volume of resulting D-brane. In this expression we implicitly presume that $\mathcal{T}(y)$ should be multiplied with some form factor that express the fact that this field is localised on world-volume of the vortex. This form factor will be the same as in case of BPS D-brane. We will write the form of this form factor in the end of calculation.

To make thinks more clear we write the action for system of N non-BPS D-branes

$$\begin{aligned}S = -C_9 \int d^{10}x \left\{ 1 + \frac{(2\pi\alpha')^2}{4} (\text{Tr} F_{MN} F^{MN} + 2i \text{Tr} \theta_L \Gamma^M D_M \theta_L + \right. \\ \left. + 2i \text{Tr} \theta_R \Gamma^M D_M \theta_R) \right\} F(T, DT, \dots)\end{aligned}\tag{4.7}$$

and the form of F function, which is present in (4.7):

$$F(T, DT..) = \frac{1}{2^k} \left[\text{Tr} D_M T D^M T + \text{Tr} (f(T) \overline{\theta}_R \theta_L) + V(T) \right]\tag{4.8}$$

When we put (4.6) into (4.8) we get

$$F = \left[\left\{ \sum_{i=1}^{2k} \left(\frac{dT_i}{dx^i} \right)^2 - m^2 T_i^2 + \lambda T_i^4 + \frac{m^2}{4\lambda} \right\} + \left\{ \partial_\mu \mathcal{T} \partial^\mu \mathcal{T} - m^2 \mathcal{T}^2 + \lambda \mathcal{T}^4 + f(\mathcal{T}) \overline{\theta}_R \theta_L \right\} \right]\tag{4.9}$$

where the second bracket is localised in the core of the vortex. In previous expressions we have used the fact that the fields $T_i(x^i)$ are zero in the point $x^i = 0$. In previous

expression the partial derivative ∂_μ means derivative with respect to the tangent coordinates of resulting non-BPS D-brane. In deriving the interaction terms between fermions and tachyon we have also used the fact that

$$f(\Gamma^{2k+1}\mathcal{T}) = \Gamma^{2k+1}f(\mathcal{T}) \quad (4.10)$$

because $f(\mathcal{T})$ is odd function of its argument. With using previous equation we have the final result for interaction term between fermions and tachyon

$$\text{Tr}f(\mathcal{T})\bar{\theta}_R\theta_L \Rightarrow \text{Tr}(\Gamma^{2k+1}\Gamma^{2k+1})f(\mathcal{T})\bar{\theta}_R\theta_L = 2^k f(\mathcal{T})\bar{\theta}_R\theta_L \quad (4.11)$$

Next calculation is the same as in previous section. With using the fact that all fields in first bracket in (4.7) are independent on x^i , we can easily make integration over these coordinates (again with appropriate insertions of convergence factor in the second bracket in (4.8)) and we obtain the action for non-BPS Dp-brane of codimension $2k$:

$$\begin{aligned} S = -2^k(0.606)^{2k}C_p \int_{R^{1,p}} d^{p+1}x \left\{ 1 + \frac{(2\pi\alpha')^2}{4} \left[F_{\mu\nu}F^{\mu\nu} + 2\partial_\mu X^i \partial^\mu X^i + \right. \right. \\ \left. \left. + \bar{\theta}_R \Gamma^\mu \partial_\mu \theta_R + \bar{\theta}_L \Gamma^\mu \partial_\mu \theta_L \right] \times \right. \\ \left. \times \left[\partial_\mu \mathcal{T} \partial^\mu \mathcal{T} + \left(-m^2 \mathcal{T}^2 + \lambda \mathcal{T}^4 + \frac{m^4}{4\lambda} \right) + f(\mathcal{T})\bar{\theta}_R\theta_L \right] \right\} \end{aligned} \quad (4.12)$$

where $p = 9 - 2k$. As in previous section, we have obtained the fields describing transverse fluctuations from the term

$$F_{\mu i} F^{\mu i} = 2\partial_\mu A^i \partial^\mu A^i = 2\partial_\mu X^i \partial^\mu X^i \quad (4.13)$$

We can also obtain the action for non-BPS D-brane of codimension $2k$ from the 2^k non-BPS D9-branes via tachyon condensation in the "step by step" construction presented in section (2) as follows:

D7-brane: This brane has codimension $2k = 2$ so that appropriate number of D9-branes is 2. Then we get following sequence:

$$2D9 \rightarrow D8 + \overline{D8} \rightarrow (2)D7 \quad (4.14)$$

where the number in the bracket (2) expresses the presence of factor 2 in front of the action. The meaning of this number has been discussed in previous section.

D5-brane: It is brane of codimension $2k = 4$ so that appropriate number of D9-branes is $2^k = 4$. The sequence of D-branes has a form:

$$4D9 \rightarrow 2D8 + 2\overline{D8} \rightarrow (2)2D7 \rightarrow (2)(D6 + \overline{D6}) \rightarrow (4)D5 \quad (4.15)$$

For non-BPS D3 and D1-brane the situation is similar.

In previous three sections we have seen that we are able to obtain action (more precisely, kinetic part of the action) for all BPS and non-BPS D-branes in Type IIA theory. In the next section we will discuss the emergence of D-branes in Type IIB theory.

5. D-branes in Type IIB theory

In this section we will discuss the emergence of BPS and non-BPS D-branes in Type IIB theory. We start with construction of BPS D-branes.

In [9] it was proposed that all BPS D-branes in Type IIB theory can arise as a tachyon topological solution in world-volume of space-time filling system of D9-branes and D9-antibranes. In [1] we have proposed the action for system of 2^{k-1} branes and antibranes in the form:

$$\begin{aligned}
S = -C_9 \int_{R^{1,9}} d^{10}x \left[1 + \frac{(2\pi\alpha')^2}{4} \left\{ \text{Tr} \left(F_{MN} F^{MN} + 2i\bar{\theta} \Gamma^M D_M \theta \right) \right. \right. \\
\left. \left. + 4i \text{Tr} \bar{\mathcal{B}} \Gamma^M \tilde{D}_M \mathcal{B} + \text{Tr} \left(F'_{MN} F'^{MN} + 2i\bar{\theta}' \Gamma^M D'_M \theta' \right) \right\} \right] \times \\
\times \frac{1}{2^k} \left\{ 2^k \frac{m^4}{4\lambda} + 2\text{Tr}(\tilde{D} T^\mu \overline{\tilde{D}_\mu T}) + (-m^2 \text{Tr}(T \bar{T}) + \lambda \text{Tr}(T \bar{T})^2) \right. \\
\left. + \text{Tr}(f(T) \bar{\mathcal{B}} \theta) + \text{Tr}_k(f(\bar{T}) \bar{\mathcal{C}} \theta') \right\}
\end{aligned} \tag{5.1}$$

where $F \in U(2^{k-1})$ is gauge field living on 2^{k-1} D9-branes, $F' \in U(2^{k-1})$ is gauge field living on 2^{k-1} D9-antibranes, θ, θ' are corresponding superpartners and tachyon T and fermionic field \mathcal{B} transform in $(\mathbf{2}^{k-1}, \overline{\mathbf{2}^{k-1}})$ of gauge group $U(2^{k-1}) \times U(2^{k-1})$.

According to ref.[9] BPS D-brane of codimension $2k$ arise from tachyon condensation on system of space-time filling 2^{k-1} D9-branes and 2^{k-1} D9-antibranes. We can ask the question whether tachyon condensation in (5.1) leads to the action of BPS D-branes. We will show on example of step by step construction that this is really true.

D7-brane This is a brane of codimension $2k = 2$. Than sequence of tachyon condensation has a form:

$$D9 + \overline{D9} \rightarrow (2)D8 \rightarrow (2)D7 \tag{5.2}$$

where the factor (2) in the second term in previous expression has the same meaning as in section (4).

D5-brane This is a brane of codimension $2k = 4$ and tachyon condensation has a form:

$$2D9 + 2\overline{D9} \rightarrow 2(2)D8 \rightarrow (2)(D7 + \overline{D7}) \rightarrow (4)D6 \rightarrow (4)D5 \tag{5.3}$$

D3-brane This is a brane of codimension $2k = 6$ and tachyon condensation has a form:

$$4D9 + 4\overline{D9} \rightarrow (2)4D8 \rightarrow (2)(2D7 + 2\overline{D7}) \rightarrow (4)2D6 \rightarrow (4)(D5 + \overline{D5}) \rightarrow (8)D4 \rightarrow (8)D3 \quad (5.4)$$

D1-brane This is a brane of codimension $2k = 8$ and tachyon condensation has a form:

$$8D9 + 8\overline{D9} \rightarrow (2)8D8 \rightarrow (2)(4D7 + 4\overline{D7}) \rightarrow (4)4D6 \rightarrow (4)(2D5 + 2\overline{D5}) \rightarrow (8)2D4 \rightarrow (8)(D3 + \overline{D3}) \rightarrow (16)D2 \rightarrow (16)D1 \quad (5.5)$$

We can also use the direct tachyon condensation presented in section (3). Let us consider the BPS D-brane of codimension $2k$. This corresponds to tachyon condensation in world-volume of 2^{k-1} D9-branes and D9-antibranes. As a first step we will make the tachyon condensation in the form of kink solution, which leads to the 2^{k-1} non-BPS D8-branes with gauge group $U(2^{k-1})$ (with the factor 2 in front of action). Since transverse space to the vortex is now $2k - 1$ dimensional, the number of non-BPS D8-branes, that are needed for construction of vortex is equal to $2^{2(k-1)/2} = 2^{k-1}$, which agrees with dimension of gauge group. Through tachyon condensation on world-volume of non-BPS D8-branes, as was presented in section (3) we get the stable BPS D-brane of codimension $2k$ (More precisely, the bound state of 2^k D-branes of codimension $2k$).

The construction presented in previous paragraph is also appropriate for construction of non-BPS D-branes in IIB theory of codimension $2k + 1$. Following [17] this brane should emerge as a tachyon vortex solution in world-volume theory of 2^k D9-branes and D9-antibranes. Following the procedure in previous paragraph, we can first form a kink solution to form 2^k non-BPS D8-branes in Type IIB theory. Then non-BPS D-brane, which has originally codimension $2k + 1$, appears as an object of codimension $2k$ in world-volume theory of non-BPS D8-brane. As we have seen in section (4) on example of construction of non-BPS D-branes in Type IIA theory (they have codimension equal to $2k$), the vortex solution, which looks like $T \sim \sum_{i=1}^{2k} \Gamma_i x^i$ leads to unstable non-BPS D-brane. We then can claim that tachyon condensation in the form in the world-volume theory of 2^k D9-branes and D9-antibranes leads to the non-BPS D-brane of codimension $2k + 1$ in Type IIB theory.

We have seen in this section that we can get the correct actions for BPS and non-BPS D-branes in Type IIB theory. In the next section we will discuss the possible form of Wess-Zumino term for non-BPS D-branes.

6. Wess-Zumino term for non-BPS D-brane

In this section we will show that tachyon condensation in Wess-Zumino term for non-

BPS D-brane leads to correct WZ term for BPS D-brane. We start from generalised form of WZ term, which is based on previous works [18, 19] and on the works [2, 4]. We hope, that this term describes correctly the coupling between non-BPS D-branes and RR forms.

We propose RR interaction for non-BPS D-branes in the form:

$$I_{WZ} = \mu_p \int C_p \wedge \text{Tr} \{ (a_1 DT + a_3 (DT)^3 + \dots + b_1 DT \wedge T^2 + \dots) \exp((2\pi\alpha')F) \} = \mu_p \sum_{k,l} I_{k,l}^A \quad (6.1)$$

where $\mu_p = \frac{2\pi}{(4\pi^2\alpha')^{\frac{p+1}{2}}}$ and where

$$I_{k,l}^A = a_{k,l} \int C \wedge \text{Tr} (DT)^{2k+1} T^{2l} \exp((2\pi\alpha')F) \quad (6.2)$$

and where $a_{k,l}$ is some numerical constant of dimension $[a_{k,l}] = l_s^{-2l}$. Unfortunately we do not know the values of these constants, but they are not important for us in this section, because we will not carry about numerical factors in this section.

We must say few words about (6.1). Firstly, we can have only odd powers of T in WZ coupling, as was explained in [2, 4]. Secondly, we have replaced ordinary derivatives with covariant derivatives as in [1]. We have included higher powers of DT in (6.1), which can be seen as a generalisation of [18] and we have introduced factors proportional to T^{2k} as in ref.[19]. In this section we will show on various examples that proposed action (6.1) correctly reproduces WZ term for D-branes in Type IIA theory.

First example is "step by step" construction on 16 non-BPS D9-branes in Type IIA theory with gauge group $U(16)$. We have seen, that this solution leads to action for single D0-brane. In this section, we apply this construction for WZ term (6.1). We take tachyon field in the form

$$T(x) = \begin{pmatrix} T_0(x)1_{8 \times 8} & T(y) \\ \overline{T(y)} & -T_0(x)1_{8 \times 8} \end{pmatrix} \quad (6.3)$$

Standard analysis leads to

$$F = \begin{pmatrix} F & 0 \\ 0 & F' \end{pmatrix} \quad (6.4)$$

Then first term gives

$$a_{1,0} \int C \text{Tr} DT \wedge e^{(2\pi\alpha')F} = 0.41 a_{1,0} (4\pi^2\alpha')^{1/2} \int C_p \wedge (e^{(2\pi\alpha')F} - e^{(2\pi\alpha')F'}) \quad (6.5)$$

simply from the fact that off-diagonal terms in derivation of tachyon do not contribute. In previous expression we have used the form of tachyon behaviour $T(x) = T_v \tanh(\frac{mx}{\sqrt{2}})$, which we have obtained in the second section (2). Integration of this function gives a factor 2. We see that the charge of resulting D-brane is about 0.41 of

value of correct charge of D-brane. We see two sources of discrepancy. Firstly, we do not know the value of constant factors $a_{k,l}$ in front of WZ term for non-BPS D-brane. Secondly, our tachyon solution is only rough approximation. However the form of the terms, which arise from tachyon condensation, suggest that tachyon condensation can lead to correct result. In the following we will not carry about numerical factors in front of the various terms. We will also write F instead of $(2\pi\alpha')F$ and we restore the factor $2\pi\alpha'$ in the end of the calculation.

The second term gives

$$\begin{aligned} & \int C \wedge \text{Tr}(DT^3 \wedge e^F) = \\ & \int C \wedge \text{Tr} \begin{pmatrix} \delta(x)dx & A \\ B & -\delta(x)dx \end{pmatrix} \wedge \begin{pmatrix} 0 & \tilde{D}T \\ \overline{\tilde{D}T} & 0 \end{pmatrix} \wedge \begin{pmatrix} 0 & \tilde{D}T \\ \overline{\tilde{D}T} & 0 \end{pmatrix} \wedge \begin{pmatrix} e^F & 0 \\ 0 & e^{F'} \end{pmatrix} = \\ & = \int_{R^{1,8}} C \wedge (\text{Tr} \tilde{D}T \wedge \overline{\tilde{D}T} e^F - \text{Tr} \overline{\tilde{D}T} \wedge \tilde{D}T e^{F'}) \end{aligned} \quad (6.6)$$

In the same way, third term gives

$$\begin{aligned} & \int C \wedge \text{Tr}(DT^5 \wedge e^F) = \int C \wedge \text{Tr}(D_x T \wedge (D_y T)^4 e^F) = \\ & = \int C \wedge \text{Tr} \begin{pmatrix} \delta(x) & 0 \\ 0 & -\delta(x) \end{pmatrix} \wedge \begin{pmatrix} 0 & (\tilde{D}T \wedge \overline{\tilde{D}T})^2 \\ (\overline{\tilde{D}T} \wedge \tilde{D}T)^2 & 0 \end{pmatrix} \wedge \begin{pmatrix} e^F & 0 \\ 0 & e^{F'} \end{pmatrix} = \\ & = \int_{R^{1,8}} C \wedge (\text{Tr}(\tilde{D}T \overline{\tilde{D}T})^2 \wedge e^F - \text{Tr}(\overline{\tilde{D}T} \wedge \tilde{D}T)^2 \wedge e^{F'}) \end{aligned} \quad (6.7)$$

Generally, we obtain the result:

$$\begin{aligned} & \int C \wedge \text{Tr}(DT)^k e^F \Rightarrow \\ & \int_{R^{1,8}} C \wedge (\text{Tr}(\tilde{D}T \wedge \overline{\tilde{D}T})^{k-1} e^F - \text{Tr}(\overline{\tilde{D}T} \wedge \tilde{D}T)^{k-1} e^{F'}) \end{aligned} \quad (6.8)$$

Next term is

$$\int C \wedge DTT^2 e^F \quad (6.9)$$

In the point $x = 0$, diagonal terms in T are zero, so we have

$$T(x=0) = \begin{pmatrix} 0 & T(y) \\ \overline{T}(y) & 0 \end{pmatrix} \Rightarrow T^2 = \begin{pmatrix} T\overline{T} & 0 \\ 0 & \overline{T}T \end{pmatrix} \quad (6.10)$$

Then we obtain from previous equation (Only derivative with respect to x participates in this expression, because the other derivatives are off-diagonal)

$$\begin{aligned}\int C \wedge \text{Tr} DTT^2 &= \int C \wedge \text{Tr} \begin{pmatrix} \delta(x) & 0 \\ 0 & -\delta(x) \end{pmatrix} \begin{pmatrix} T\bar{T} & 0 \\ 0 & \bar{T}T \end{pmatrix} e^F = \\ &= \int_{R^{1,8}} C \wedge (\text{Tr} T\bar{T} \wedge e^F - \text{Tr} \bar{T}T \wedge e^{F'})\end{aligned}\tag{6.11}$$

In the same way we obtain:

$$\begin{aligned}\int C \wedge \text{Tr} DTT^{2k} e^F &\Rightarrow \\ \int_{R^{1,8}} C \wedge (\text{Tr}(T\bar{T})^k e^F - \text{Tr}(\bar{T}T)^k e^{F'})\end{aligned}\tag{6.12}$$

Next term is

$$\int C \wedge \text{Tr}(DT)^3 T^2 \wedge e^F\tag{6.13}$$

which leads to

$$\begin{aligned}\int C \wedge \text{Tr} \begin{pmatrix} \delta(x) & A \\ B & -\delta(x) \end{pmatrix} \wedge \begin{pmatrix} \tilde{D}T \wedge \overline{\tilde{D}T} & 0 \\ 0 & \overline{\tilde{D}T} \wedge \tilde{D}T \end{pmatrix} \wedge \begin{pmatrix} T\bar{T} & 0 \\ 0 & \bar{T}T \end{pmatrix} e^F = \\ = \int C \wedge (\text{Tr} \tilde{D}T \wedge \overline{\tilde{D}T} T\bar{T} e^F - \overline{\tilde{D}T} \wedge \tilde{D}T \bar{T}T e^{F'})\end{aligned}\tag{6.14}$$

Generally, we obtain via tachyon condensation in form a kink solution:

$$\begin{aligned}I_{k,l}^A &= \int C \wedge (DT)^{2k+1} T^{2l} e^F \Rightarrow \\ \int_{R^{1,8}} C \wedge (\text{Tr} \tilde{D}T \wedge \overline{\tilde{D}T})^k (T\bar{T})^l \wedge e^F - \text{Tr} \overline{\tilde{D}T} \wedge \tilde{D}T (\bar{T}T)^l \wedge e^{F'} &= I_{k,l}^B\end{aligned}\tag{6.15}$$

and generalised WZ term for system of D8-branes and D8-antibranes is

$$I_{WZ}^B = \mu_8 \sum_{k,l} I_{k,l}^B\tag{6.16}$$

We can see striking similarity with result in [19]³.

³We will write in each step the factor μ_p , because in each step we obtain factor $(\alpha')^{1/2}$. As was explained above, we omit the other numerical factors. We also freely use the symbol of delta function, in order to express the fact that various fields are localised only in the core of the vortex. We have discussed the meaning this delta function in previous sections.

Now we construct kink solution on the world-volume of 8-branes and 8-antibranes. This solution was given as:

$$T(x, y) = T_v(x)1_{8 \times 8} + iT(y)\delta(x), \quad T(y)^\dagger = T(y) \quad (6.17)$$

We have seen, that this solution gives correct kinetic term for non-BPS D7-brane. From this analysis we know that $F = F'$. We start our analysis with the terms $I_{k,0}$. We will write $(\tilde{D}T \wedge \overline{\tilde{D}T})^k = (\tilde{D}T \overline{\tilde{D}T}) \wedge (\tilde{D}T \wedge \overline{\tilde{D}T})^{k-1}$. Now we use the fact, that in second bracket in previous expression we do not have derivation with respect x , so we obtain $\tilde{D}T = iDT$, $\overline{\tilde{D}T} = -iDT$ so that the second bracket is equal to

$$(DT \wedge DT)^{k-1} = (DT)^{2k-2}$$

and the first bracket is equal to

$$dT \wedge \tilde{D}\overline{T} = d(T\tilde{D}\overline{T}) = -id(TDT)$$

where we have used the fact that all massless fields as well as $T(y)$ are independent on x . For expression $(\overline{\tilde{D}T} \wedge \tilde{D}T)^k = (\overline{\tilde{D}T} \wedge \tilde{D}T) \wedge (\overline{\tilde{D}T} \wedge \tilde{D}T)^{k-1}$ we can do the same analysis. The second bracket is equal to $(DT)^{2k-2}$ and the first bracket leads to the result

$$\overline{\tilde{D}T} \wedge DT = -DT \wedge \overline{\tilde{D}T} = id(TDT)$$

Finally, we obtain

$$I_{k,0} = -2i \int_{R^{1,8}} C \wedge d(\text{Tr} T (DT)^{2k-1} e^F) = -2i \int_{R^{1,7}} C \wedge \text{Tr} T DT^{2k-1} e^F \quad (6.18)$$

where we have made integration over x . Previous expression is a correct result (up the sign $(-2i)$) for non-BPS D-branes. The same analysis can be used for general $I_{k,l}$, because the only difference is in presence of term $T\overline{T} = (iT)(-iT) = T^2$, where we have used the fact, that in point $x = 0$, T_0 is zero. So that we obtain general result:

$$I_{k,l}^B \Rightarrow -2i \int_{R^{1,7}} C \wedge \text{Tr} (DT)^{2k-1} T^{2l} e^F = (-2i) I_{k-1,l}^A \quad (6.19)$$

The whole WZ term is a term appropriate for non-BPS D-brane, up the factor $(-2i)$:

$$I_{WZ} = (-2i) \mu_7 \sum_{k,l} I_{k,l} \quad (6.20)$$

We can again construct kink solution on non-BPS D7-brane with gauge group $U(8)$, leading to the action for 4 D6-branes and 4 D6-antibranes⁴. Following general recipe (6.8)

$$I_{k-1,l}^A \Rightarrow I_{k-1,l}^B \Rightarrow I_{WZ}^A \Rightarrow I_{WZ}^B \quad (6.21)$$

⁴We can notice, that in front of WZ term is a factor 2. We will see, that in front of WZ term for D5-brane will be factor 4, for D3-brane will be factor 8 and finally for D1-brane the factor 16 will be present. The interpretation of these factors is the same as in section (2).

We must remember, that now the gauge group is $U(4) \times U(4)$.

Further kink solution on world-volume of brane antibrane leads to

$$I_{k-1,l}^B \Rightarrow (-2i)I_{k-2,A}, I_{WZ}^B \Rightarrow (-2i)^2 I_{WZ}^A \quad (6.22)$$

It is important to stress, that

$$I_{0,l}^B \Rightarrow 0 \quad (6.23)$$

due to the fact, that $F = F'$, $T = \bar{T}$. As a result, we obtain action for 4 non-BPS D5-branes with gauge group $U(4)$. Further kink solution leads to 2 D4-branes and 2-D4-branes with gauge group $U(2) \times U(2)$ and

$$I_{k-2,l}^A \Rightarrow I_{k-2,l}^B \quad (6.24)$$

Further kink solution leads to the WZ term for 2 non-BPS D3-branes with gauge group $U(2)$ and

$$I_{k-2,l}^B \Rightarrow (-2i)I_{k-3,l}^A, I_{WZ}^B \Rightarrow (-2i)^3 I_{k-3,l} \quad (6.25)$$

Next step gives action for D2-brane and D2-antibrane

$$I_{k-3,l}^A \Rightarrow I_{k-3,l}^B \quad (6.26)$$

Kink solution in this system gives non-BPS D-brane. In this step, covariant derivative for brane+antibrane system is

$$\tilde{D}T = dT + AT - TA' \Rightarrow dT$$

As a result, we obtain WZ term for single non-BPS D1 brane

$$I = (-2i)^4 \mu_1 \int_{R^{1,1}} C \wedge dT \sum_{l=0} T^{2l} e^F \quad (6.27)$$

Now we consider the last tachyon condensation. It is important to stress, that only term with $l = 0$ is nonzero, because other terms are zero in point $x = 0$ due to the fact, that $T(0) = 0$. Then we obtain standard result

$$I_{WZ} = 2^4 \mu_0 \int dt C_1 \quad (6.28)$$

We see, that this is a correct coupling of 16 D0-branes to RR one form so together with result in section (2) we obtain right action for BPS bound state of 16 D0-branes in IIA theory. In the next paragraph we will discuss the tachyon condensation in the form of vortex solution given in (3).

In this part we will consider construction of general Dp-brane of codimension $2k = 1$ in Type IIA theory with using gauge theory living on 2^k non-BPS D9-branes. We consider situation, when tachyon condensation leads to D-brane of codimension $2k + 1$. Following general recipe given in [10], this brane can be constructed from 2^k

non-BPS D9-branes with gauge group $U(2^k)$. For kinetic term, we have obtained correct expression in section (3). Now we turn to the problem of tachyon condensation in the term given in (6.1),(6.2):

$$I_{WZ} = \mu_9 \sum_{k,l} I_{k,l}^A \quad (6.29)$$

where

$$I_{k,l}^A = \int C \wedge \text{Tr}(DT)^{2k+1} T^{2l} e^F \quad (6.30)$$

We know that BPS D-brane of codimension $2k+1$ in Type IIA theory arises from tachyon condensation in the form

$$T(x) = \sum_{i=1}^{2k+1} \Gamma_i T(x^i) \quad (6.31)$$

We have analysed the form of this solution in (3). We know that all $T(x^i)$ must be localised in the points $x^j = 0, j \neq i$ and that $T(x^i)$ has a form $T(x^i) = T_v \tanh(\frac{mx}{\sqrt{2}})$. Derivation of previous function is $\frac{m}{\sqrt{2}}(1 - \tanh^2(\frac{mx}{\sqrt{2}}))$, which has the properties similar to delta function (more precisely, in zero slope limit $\alpha' \rightarrow 0$ is equal to zero almost everywhere and is finite in the point $x = 0$).

We immediately can see from (6.30) that only term $I_{2k+1,0}$ contribute to the form of resulting D-brane. The other terms are zero either from the fact that contain more than $2k+1$ covariant derivatives or less and than $2k+1$, so that they cannot form the volume form in the transverse space, or contain the powers of T , which are zero in the core of the vortex. As a result we obtain from $I_{2k+1,0}$:

$$I_{2k+1,0} = \mu_9 \int_{R^{1,9}} C \wedge \text{Tr}(DT)^{2k+1} e^{(2\pi\alpha')F} \Rightarrow 2^k \mu_p \int_{R^{1,p}} C \wedge e^{(2\pi\alpha')F} \quad (6.32)$$

up to possible numerical factor. The factor 2^k comes from the trace and field strength F corresponds to abelian gauge field, as in (3). The emergence of the factor 2^k again suggest that resulting configuration is in fact the bound state of 2^k D-branes of codimension $2k+1$. We have discussed this issue in section (3).

Of course, as in sections (3), we can consider direct construction of WZ term for BPS D-brane of codimension $2k+1$ in the world-volume of 16 non-BPS D9-branes. Consider Dp-brane of codimension $2k+1$. We know from section (3), that gauge field has a form

$$F = 1 \otimes \mathcal{F} \quad (6.33)$$

where $\mathcal{F} \in U(2^{k-4})$. We also know, that only term with one covariant derivative DT contributes to the Wess-Zumino term, which has a form:

$$I = \mu_9 \int C \wedge \text{Tr}(DT)^{2k+1} \exp((2\pi\alpha')F) \quad (6.34)$$

We know that covariant derivative has a form $DT = dt(x) \otimes 1_{2^{4-k} \times 2^{4-k}}$, where $t(x)$ has the same form as in previous paragraph. With using the formula $\text{Tr}(A \otimes 1)(1 \otimes B) = \text{Tr}A \text{Tr}B$ we get immediately the result (up to possible numerical factor):

$$I_{WZ} = 2^k \mu_p \int C \wedge \text{Tr} \exp((2\pi\alpha')\mathcal{F}) \quad (6.35)$$

where Tr in previous expression goes over fundamental representation of $U(2^{4-k})$. We again see the factor 2^k in front of action, which suggests that each D-brane of codimension $2k+1$ in resulting configuration is in fact the bound state of 2^k D-branes.

We can also discuss the emergence of WZ term for non-BPS D-brane of codimension $2m$. When we use the step by step construction proposed in section (6) we obtain immediately the WZ term for non-BPS D-brane. We can also start with configuration given in (3) for construction of non-BPS D-branes. Again only term with $I_{2k+1,l}$, $2k+1 > 2m$ contribute in this construction. The terms with more covariant derivative are nonzero, due to the fact that the additional covariant derivatives contain the derivation of tachyon field \mathcal{T} . We see that we have also nonzero terms with various powers of T^{2l} . This is a consequence of the fact that in the core of the vortex the unconstrained tachyon field is nonzero. With using $T(y)^{2l} = \mathcal{T}(\Gamma_{2k+1})^{2l} = \mathbf{1}\mathcal{T}$, we immediately obtain the action for non-BPS D-brane of codimension $2k$

$$\begin{aligned} I_{k,l} &= \mu_9 \int C \wedge DT^{2k+1} T^{2l} e^{(2\pi\alpha')F} \Rightarrow \\ I_{k-m,l} &= 2^k \mu_p \int C \wedge (dT)^{2k+1-2m} \mathcal{T}^{2l} e^{(2\pi\alpha')F} \end{aligned} \quad (6.36)$$

For D-branes in Type IIB theory the situation is the same as in section (5). For BPS D-brane of codimension $2k$ we start with WZ term for system of 2^{k-1} D9-branes and D9-antibranes. Then we can proceed in the same way as in (6) and we will finish with WZ term for BPS D-brane. Equivalently, we could construct the unstable D8-brane with gauge group $U(2^{k-1})$ and than proceed in the same way as in previous paragraph. For non-BPS D-branes the situation is basically the same.

7. Conclusion

In previous sections we have seen on many examples that our approach to the problem of tachyon condensation gives correct form of action for D-branes in Type IIA and Type IIB theory. Of course, more direct calculation should be needed for confirming our result, especially interesting appears to us approach presented in ref.[24, 25, 28, 29]. We know that form of our action for non-BPS D-brane is rather simple approximation, which should be supported by more direct calculation in string theory. On the other hand, success of our approach allows us to claim, that even with this simple form of action we are able to obtain correct form of action for BPS

D-branes. It is possible that in heart of the success lies BPS property of D-branes. It would be very nice to confirm our calculation with direct method as in ref.[24, 25].

We would like also see the direct relation to the K-theory [9, 10]. Analysis of D-branes in K-theory is based on nontrivial gauge fields that live on non-BPS D-branes or on system of D-branes and D-antibranes. On the other hand we have seen that in all our situations the gauge fields are trivial. We expect that the nontrivial behaviour of gauge field emerges from more general form of solution of tachyon field. We hope to return to this question in the future.

It would be interesting to study the other theories, especially Type I theory and M -theory, following [16]. It would be also interesting to study tachyon condensation in the other systems, following [26, 27]. And finally, it would be interesting to study the problem of emergence of non-Abelian gauge symmetry for system of N D-branes, that arise from tachyon condensation.

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References

- [1] J. Klusoň, "*D-branes from N non-BPS D9-branes in IIA theory*", *J. High Energy Phys.* **0002** (017) 2000, [hep-th/9910241](#).
- [2] A. Sen, "*Supersymmetric World-volume Action for Non-BPS D-branes*", [hep-th/9909062](#).
- [3] A. A. Tseytlin, "*Born-Infeld action, supersymmetry and string theory*", [hep-th/9908105](#).
- [4] A. Sen, "*Non-BPS states and Branes in String Theory*", [hep-th/9904207](#).
- [5] A. Lerda and R. Russo, "*Stable Non-BPS states in String theory: A Pedagogical Review*", [hep-th/9905006](#).
- [6] J. Schwarz, "*TASI Lectures on Non-BPS D-Branes Systems*", [hep-th/9908144](#).
- [7] A. Sen, " *$SO(32)$ Spinors of Type I and other solitons on Brane-Antibrane Pair*", *J. High Energy Phys.* **9809** (023) 1998, [hep-th/9808141](#).
- [8] A. Sen, "*Type I D particle and its Interactions*", *J. High Energy Phys.* **9810** (021) 1998, [hep-th/9809111](#).
- [9] E. Witten, "*D-Branes and K theory*", [hep-th/9810188](#).
- [10] P. Hořava, "*Type IIA D-Branes, K-Theory and Matrix theory*", [hep-th/9812135](#).
- [11] K. Olsen and R. J. Szabo, "*Brane Descent Relations in K theory*", [hep-th/9904157](#).
- [12] K. Olsen and R. J. Szabo, "*Constructing D-Branes From K theory*", [hep-th/9907140](#),
- [13] A. Sen, "*Tachyon condensation on Brane-Antibrane system*", *J. High Energy Phys.* **08** (012) 1998, [hep-th/9805170](#).
- [14] A. Sen, "*Stable non-BPS states of BPS D-particles*", *J. High Energy Phys.* **08** (010) 1998.
- [15] A. Sen, "*BPS D-branes on non-supersymmetric cycles*", *J. High Energy Phys.* **12** (021) 1998, [hep-th/9812031](#).
- [16] P. Hořava, "*M theory as a holographic theory*", *Phys. Rev. D* **59** (046004) 1999.
- [17] J. A. Harvey, P. Horava and P. Kraus, "*D-Sphalerons and the Topology of String Configurations Space*", [hep-th/0001143](#)
- [18] M. Billo, B. Craps and F. Rosse, "*Ramond-Ramond coupling of non-BPS D-branes*", [hep-th/9905157](#).
- [19] C. Kennedy and A. Wilkins, "*Ramond-Ramond Coupling on Brane-Antibrane Systems*", [hep-th/9905185](#).

- [20] E. Witten, "*Bound States of Strings and p-Branes*", *Nucl. Phys. B* **460** (1996) 335, [hep-th/9510135](#).
- [21] W. Taylor IV., "*Lectures on D-branes, Gauge Theory and M(atrices)*", [hep-th/9801192](#).
- [22] M. Green, J. H. Schwarz and E. Witten, "*Superstring theory, Vol.2*", Cambridge University Press 1987.
- [23] W. Taylor IV, "*Adhering 0-branes to 6-branes and 8-branes*", [hep-th/9705116](#).
- [24] A. Sen, "*Universality of the Tachyon Potential*", [hep-th/9911116](#).
- [25] A. Sen and B. Zwiebach, "*Tachyon Condensation in String Field Theory*", [hep-th/9912249](#).
- [26] E. Bergshoeff, E. Eyras, R. Halbersma, C. M. Hull, Y. Lozano and J. P. van der Schaar, "*Space-time-filling Branes and Strings with Sixteen Supercharges*", [hep-th/9812224](#).
- [27] L. Houart and Y. Lozano, "*S-Duality and Brane Descent Relation*", [hep-th/9911173](#).
- [28] N. Berkovits, A. Sen and B. Zwiebach, "*Tachyon Condensation in Superstring Theory*", [hep-th/0002211](#)
- [29] N. Berkovits, "*The Tachyon Potential In Open String Field Theory*", [hep-th/0001084](#)